Making (Non–Standard) Choices

by

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ABSTRACT

In this paper we consider non–standard choices, choices that violate contraction and/or expansion consistency conditions such as the weak axiom of revealed preference. The choices that we discuss and describe axiomatically are, among others, the rational shortlist method which is a two–stage procedure, the choice of the second largest option as the best element, the choice of the median object and choices according to an attention ordering. The various characterizations show that there is not much of an overlap among the different axiom systems. In other words, non–standard choices do not seem to have a uniform structure. This is due to the fact that the underlying norms can vary considerably.

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1 Introduction

Imagine a person who, given three alternatives x, y and z, chooses x over y, y over z, and z over x. This person's choices definitely are not transitive. What we encounter instead is a case of cyclical choice. There have been quite a few experiments which reveal that a non–negligible number of individuals exhibits cycles of binary choice, for one reason or another.[1] Simple mistakes but also a high degree of complexity of the alternatives at stake have frequently been given as reasons for cyclical choice behaviour.

A constituent element of the standard model of rational choice is the "weak axiom of revealed preference" WARP (Samuelson, 1938) which says that if some alternative x is picked when another alternative y is available, then y is never chosen from a set of alternatives including both x and y. The WARP axiom is equivalent to the following requirement: If for two sets X and Y with $X \subset Y$, $C(Y) \cap X$ is nonempty, where $C(Y)$ is the choice from Y, then the choice from X is $C(X) = C(Y) \cap X$ (Arrow, 1959).

There is wide agreement that WARP, together with its stronger version, the "strong axiom of revealed preference" (Houthakker, 1950), and Arrow's requirement are the central consistency conditions of economic behaviour. Individuals who satisfy either of these requirements are viewed as acting "fully rationally". The question then arises how one should describe the behaviour of those agents who violate these conditions. Are they behaving irrationally?

Sen (1993) has argued that a violation of WARP or Arrow's condition is by no means sufficient to claim that an agent's choices have to be viewed as irrational. Sen writes that "we cannot determine whether the person is failing in any way without knowing what he is trying to do, that is, without knowing something external to the choice itself" (1993, p. 501). Sen makes a distinction between purely internal grounds which are confined to the choice itself, and the environment or context within which the choice act has taken place. Here, motivations, goals, and principles play a role and only when this

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[1]In an experiment of choice behaviour among gray jays, Waite (2001), reported in Manzini and Mariotti (2007), finds that all the birds preferred option a to b and b to c, but no bird preferred a to c, where all alternatives, characterized by (n,l), consisted in getting n raisins at the end of an l cm long tube, with a = (1 raisin; 28 cm), b = (2;42) and c = (3;56). Apparently, many birds showed an intransitive choice behaviour in this experiment.

"external reference" has been specified, does it become reasonable to discuss the issue of rational versus irrational behaviour. In the conventional case or the case that standard microeconomics textbooks discuss, choices are induced by preference optimization and this external reference provides justification for the conditions described above.

Let us briefly mention some other cases; we shall discuss them in more detail in the main body of this paper. Consider the following situation depicted by Manzini and Mariotti (2007). Let there be three alternatives a, b and c. Suppose that c Pareto dominates a while no other comparisons are possible according to the Pareto criterion. Assume now that the choosing individual considers a to be fairer than b and b fairer than c. The individual decides first according to the Pareto principle and only then, when Pareto is not decisive, according to fairness. Consequently, the individual's choice function C is such that $C(\{a, b, c\}) = \{b\}$. First a is eliminated by c due to the Pareto principle and then c is eliminated by b due to the fairness criterion. As the reader can see, two criteria are consecutively applied in this choice situation.

If we consider binary choices, we obtain $C(\{a, b\}) = \{a\}$ and $C(\{b, c\}) = \{b\}$ due to fairness and $C(\{c, a\}) = \{c\}$ due to Pareto. So we get a pairwise cyclical choice pattern as in our very first example and also a violation of WARP since a was chosen against b while b was picked from a set where both a and b were present. Or, according to Arrow's choice axiom, b was chosen from the superset $\{a, b, c\}$ but not picked from the subset $\{a, b\}$. Can the described choice behaviour be termed irrational? Obviously not though it violates the standard axioms of rational choice, i.e., WARP and Arrow's axiom of choice consistency.

Sen (1993) and Baigent and Gaertner (1996) considered the choice of the second largest element as the most preferred choice. So let there be three pieces of cake with a being largest and c being smallest with b lying in between. Then an individual who has internalized the above principle will choose c from $\{a, c\}$ and c from $\{b, c\}$ but will pick b from $\{a, b, c\}$. Such choice behaviour violates Arrow's choice axiom and also a condition called expansion. The latter says that if an alternative x is chosen from a set S and from a set T, then it must also be chosen from $S \cup T$. Again, do we have an instance of irrational choice in front of us? The person who follows the principle of choosing the second largest and not the largest element would clearly deny this.

Finally, let us consider a person who always picks the median element (see Gaertner and Xu, 1999). So let there be seven elements a, b, c, d, e, f and g arranged according to their price from highest to lowest. The individual chooses c from $\{a, b, c, d, e\}$ and picks c from $\{a, b, c, f, g\}$ but chooses d from $\{a, b, c, d, e, f, g\}$. The reader realizes immediately that Arrow's choice axiom is again violated and also the requirement that we called expansion is not fulfilled. Again, does the individual described show any trace of inconsistency?

The purpose of this paper is twofold. First, as has become "visible" above, we argue that choice situations that do not fulfil the standard rationality conditions by no means imply that individuals are behaving irrationally. On the contrary, they are very rational, following particular external references. Second, if the standard requirements are not satisfied as shown above, the question is which the conditions are that characterize various types of non–standard behaviour. In other words, how do the axioms look like that entail such types of choice behaviour?

In section 2 we shall discuss in more detail the three choice functions briefly introduced above. Note again that they all violate the standard axioms of rational choice. In section 3, we depict other choice functions that are non–standard. Section 4 offers some concluding remarks.

2 Three Particular Choice Functions

2.1 Sequential Rationalizability

Manzini and Mariotti (2007) have proposed a choice function that they call a rational shortlist method which works as follows. It is assumed that a decision maker uses sequentially two rationales to discriminate among the given alternatives. These rationales are applied in a fixed order, independently of the set of available alternatives, to remove inferior alternatives. The authors assert that this procedure sequentially rationalizes a choice function if, for any feasible set, "the process identifies the unique alternative specified by the choice function" (2007, p. 1824). An example of such a procedure has already been given in the introduction, the first rationale being the Pareto principle, the second rationale being the fairness criterion.

Let us become more formal now. Let X be a set of alternatives, with at least three elements. Given $S \subseteq X$ and an asymmetric binary relation $P \subseteq X \times X$, let us denote the set of P-maximal elements of S by $\max(S; P) = \{x \in S : \nexists\, y \in S \text{ with } (y, x) \in P\}$. Let χ be the set of all nonempty subsets of X. A choice function on X picks one alternative from each possible element of χ. In other words, it is a function $C : \chi \to X$ with $C(S) \in S$ for all $S \in \chi$. In the standard model of rational choice, a choice function C maximizes according to an acyclic binary relation P such that $C(S) = \max(S; P)$ for all $S \in \chi$. The new concept by Manzini and Mariotti uses two asymmetric binary relations P_1 and P_2 from $X \times X$ in order to eliminate alternatives via two sequential rounds.

Definition 1 (Manzini and Mariotti, 2007). A choice function C is a rational shortlist method (RSM) whenever there exists an ordered pair (P_1, P_2) of asymmetric relations, with $P_i \subseteq X \times X$ for $i \in \{1, 2\}$, such that
$C(S) = \max(\max(S; P_1); P_2)$ for all $S \in \chi$.
(P_1, P_2) are said to sequentially rationalize C. Each P_i is called a rationale.

In the first round the decision maker retains only those elements that "survive" according to rationale P_1. In the second round, he retains only the element that is maximal according to rationale P_2. And this is the final choice. The authors emphasize that it is crucial that the rationales and the sequence in which they are applied are invariant with respect to the choice set. Note that in the authors' example with the Pareto condition and the fairness requirement, discussed in the introduction, the outcome of the two–stage procedure would be different if the fairness criterion had been applied first.

We now introduce a more formal definition of the weak axiom of revealed preference.

Definition 2 – WARP. If an alternative x is chosen when y is available then y is not chosen when x is available. More formally, for all $S, T \in \chi : [\{x\} = C(S), y \in S, x \in T] \rightarrow [\{y\} \neq C(T)]$.

WARP is a necessary and sufficient condition such that choice is rationalized by an ordering (a complete and transitive binary relation). Note again that WARP is violated both by the criterion of choosing the second largest element and by the method of picking the median element, and is not satisfied by RSM either.

Definition 3 – Weak WARP (Manzini and Mariotti, 2007). If an alternative x is chosen both when only y is also available and when y and other alternatives $\{z_1, \ldots, z_k\}$ are available, then y is not chosen when x and a subset of $\{z_1, \ldots, z_k\}$ are available. Formally, for all $S, T \in \chi : [\{x, y\} \subset S \subset T, \{x\} = C(\{x, y\}) = C(T)] \rightarrow [\{y\} \neq C(S)]$.

Note that the maxim of picking the second largest element satisfies Weak WARP. However, the choice of the median violates this condition. This can be seen from the following example. Let there be seven elements q, v, w, x, y, z, and r with q having the highest price and r having the lowest. Let us further assume, without loss of generality, that in the case of only two objects, the higher priced object is chosen. Then $C(\{x, y\}) = \{x\}, C(\{q, v, w, x, y, z, r\}) = \{x\}$ but $C(\{w, x, y, z, r\}) = \{y\}$.

Let us next formally define the expansion condition which was already described in the introduction.

Definition 4 – Expansion. An alternative which is picked from each of two sets is also chosen from their union. Formally, for all $S, T \in \chi : [\{x\} = C(S) = C(T)] \rightarrow \{x\} = C(S \cup T)]$.

The following result now holds.

Result 1 *(Manzini and Mariotti, 2007). Let X be any (not necessarily finite) set. A choice function C on X is an RSM iff it satisfies Expansion and Weak WARP.*

As already mentioned, both the principle of picking the second largest element and the principle of choosing the median do not satisfy the expansion condition. Both rules focus on positions and therefore, the choice may change when the menu changes. Picking the largest element or the smallest element as the best choice would always satisfy Expansion.

Consider the following weakening of the expansion condition. Manzini and Mariotti call it "always chosen".

Definition 5 – Always Chosen. If an alternative is chosen in pairwise choices over all other alternatives in a set, then it is chosen from the set. Formally, for all $S \in \chi : [\{x\} = C(\{x, y\})$ for all $y \in S - \{x\}] \rightarrow [\{x\} = C(S)]$.

A binary cyclical choice pattern was manifest in the example with the two criteria of Pareto and fairness. The following property excludes such cycles.

Definition 6 – No Binary Cycles. There are no pairwise cycles of choice. Formally, for all $x_1, \ldots, x_{n+1} \in X : [C(\{x_i, x_{i+1}\}) = \{x_i\}, i \in \{1, \ldots, n\}] \rightarrow [\{x_1\} = C(\{x_1, x_{n+1}\})]$.

Manzini and Mariotti assert that the class of choice functions that do not satisfy WARP can be classified in the following way. There are three subclasses: choice functions that violate exactly one of No Binary Cycles or Always Chosen, and those that violate both. Therefore, the following result can be formulated.

Result 2: *A choice function that violates WARP also violates Always Chosen or No Binary Cycles.*

The rational shortlist method can be generalized to more than two rationales. Manzini and Mariotti call this sequential rationalizability.

Definition 7 (Manzini and Mariotti, 2007). A choice function C is sequentially rationalizable whenever there exists an ordered list $P_1, \ldots, P_k$ of asymmetric relations, with $P_i \subseteq X \times X$ for $i \in \{1, \ldots, k\}$, such that, defining recursively,

$M_o(S) = S, M_i(S) = \max(M_{i-1}(S; P_i)), i \in \{1, \ldots, k\}$, we have $C(S) = M_k(S)$ for all $S \in \chi$.

We say that $(P_1, \ldots, P_k)$ sequentially rationalize C; each P_i is called a rationale.

For each S, there are sequential rounds of elimination of alternatives. At each round, only those elements that are maximal according to a round–specific rationale carry over to the next round. Again, the rationales and the sequence are invariant with respect to the choice set.

Result 3 *(Manzini and Mariotti, 2007). If a choice function is sequentially rationalizable, it satisfies Always Chosen.*

The intuition behind this result is very simple. If an alternative x, let's say, survives in binary contests on each stage i, given rationale $P_i, i \in \{1, \ldots, k\}$, then it eliminates all the other alternatives and is eliminated by no other rationale. So it is always chosen.

Since WARP is violated if there is a binary cycle, the following result follows from Results 2 and 3.

Result 4 *(Manzini and Mariotti, 2007). A sequentially rationalizable choice function violates WARP iff it exhibits binary cycles.*

Manzini and Mariotti call the violations of Always Chosen and No Binary Cycles two elementary pathologies of choice to which "*all* violations of "rationality" can be traced back" (2007, p. 1832). Since the principle of choosing the second largest element as well as the maxim of picking the median element both violate Always Chosen, they cannot be sequentially rationalizable. Furthermore, they reveal an elementary pathology of choice according to the two authors. Do they really? We shall look at them more closely in the following two sections.

2.2 Picking the Second Largest

Baigent and Gaertner (1996) were the first to characterize axiomatically the choice of the second largest element (or piece of cake, if you wish), thereby following an example by Sen (1993) expressing a norm of politeness or the attitude of not being too greedy. Let q denote a linear order on X; q can be seen as representing the relevant "quality"–ordering over the given set of alternatives. In other words, q may represent an ordering in terms of size or length (from highest to lowest, let's say), in terms of a specified quality such as speed or acceleration, and so on. In contrast to the authors' original approach and for reasons of simplification, we only consider objects of unequal size or length, for example. Therefore, a linear ordering (complete, transitive and asymmetric) will do. Now in order to characterize the choice of the second largest, let $M(S,q)$ denote the maximal elements of $S \in K$ according to q, where K is the set of all subsets of X, including the empty set. So in contrast to the analysis in section 2.1, we shall also consider situations where the choice set will be the empty set. We shall be more specific in due course. Note that according to our assumption above, $M(S,q)$ is always unique. The standard approach of optimization would now postulate that for all $S, C(S) = M(S,q)$. The Baigent–Gaertner–Sen choice function in contrast is, for all $S \in K$,

$$C(S) = M(S - M(S,q), q). \qquad (*)$$

This means that if there are m elements arranged by q according to size, let's say, the maximal element is deleted from S and in a second step, the maximal element is chosen from the remaining $m-1$ objects. Note that according to its construction, $C(S)$ is the empty set for one–element sets and $C(\emptyset) = \emptyset$.

The characterization that we now present is not the original one from the Baigent–Gaertner article but a more recent one proposed by Xu (2007). It is somewhat simpler and also differs from the earlier one in so far as choice sets can be empty, as mentioned above. Here are the axioms.

Definition 8 – Emptiness of Singleton Choice Situations (ESCS).
For all $x \in X, C(\{x\}) = \emptyset$.

Definition 9 – Non–Emptiness of Non–Singleton Choice Situations (NENCS). For all $S \in K$ with $\sharp S \geq 2, C(S) \neq \emptyset$.

Definition 10 – Constrained Contraction Consistency (CCC). For all $S \in K$ with

$\sharp S \geq 3$, there exists $s^* \in S$ with $\{s^*\} \neq C(S)$ such that, for all $S_1, S_2 \subseteq S$, if $s^* \in S_1 \subseteq S_2$ and $C(S_2) \subseteq S_1$, then $C(S_1) = C(S_2)$.

Definition 11 – Anti–Expansion (AE). For all distinct $x, y, z \in X$, if $\{x\} = C(\{x, y\}) = C(\{x, z\})$, then $\{x\} \neq C(\{x, y, z\})$.

Definition 12 – Consistency of a Revealed Norm (CRN). For all $S \in K$ and all $x \in S$, if $\{x\} \neq C(S)$ and $\{x\} \neq C(\{x \cup C(S)\})$, then, for all $y \in S - \{x\}$, $\{x\} \neq C(\{x, y\})$.

Let us give some brief explanations of the five axioms above. ESCS just states that the choice from single–element choice situations is "choosing nothing" or abstaining from picking the only element that is given. It may be another expression of politeness. This is in conformity with Sen's (1993) example of not choosing the last apple in the fruit basket. Where there is more than one apple left, NENCS says that a "genuine" choice will be made.

Axiom CCC is a contraction consistency condition in the vein of Arrow's (1959) requirement of choice consistency, though weaker. It says that if there is some focal alternative s^* that is not chosen from the grand set (for example, the largest object), and this alternative is also included in subsets S_1 and S_2, then the choice from S_2, if it is contained in S_1, is also the choice from S_1. Note that the latter part of this argument is what Arrow required in his consistency condition.

Definition 11, i.e. Anti–Expansion, is the opposite of Expansion in definition 4, here defined in a binary way. An element that is picked in all binary contests is not chosen from the union of these elements. Finally, axiom CRN requires that if an element x is not picked from set S, neither from a set which consists of x and the chosen element from S, then x should not be chosen in any pairwise contest between itself and any element from S.

Here is Xu's (2007) characterization.

Result 5. *A choice function C that reflects the norm of never picking the largest element is in this sense rationalizable iff it satisfies axioms ESCS, NENCS, CCC, AE and CRN.*

Given the Baigent–Gaertner–Sen choice function in $(*)$, any choice function C on X is rationalizable according to $(*)$ iff there exists an ordering q on X that satisfies $(*)$.

Does the choice function just characterized show any signs of elementary pathology? If the expansion condition or its weakening, viz. Always Chosen, are viewed in such a way that any violation of either of these two requirements is pathological, then the answer should definitely be "yes". On the other hand, the norm which is behind the maxim of never choosing the largest element as the first best choice is very clear and intuitive, though not every agent is, of course, supposed to follow this rule. The anti–expansion axiom mirrors a fundamental feature of this choice behaviour, viz. that the individual position of objects within an overall arrangement of these objects counts. The addition as well as subtraction of alternatives matters a lot, and does change an individual's choice, a feature that will also become manifest in the third choice function that we shall discuss now.

2.3 The Choice of the Median Element

Gaertner and Xu (1999) offered a characterization of the choice of the median as the best alternative. Picking the median makes a lot of sense in various contexts. Choosing a median–priced gift manifests a good balance between the extremes of appearing as a miser and showing off. The pursuit of balancedness can be found in classic Chinese philosophy. However, as mentioned at the end of our introduction, picking the median–sized element violates Arrow's consistency requirement and Expansion.

Let us again become more formal. We assume that q once more represents the relevant "quality"–ordering over a given set of alternatives. For example, q linearly orders the party spectrum from left to right (or right to left, if preferred); q may order objects according to their price or size or weight.

For all $S \in \chi$, the set of all nonempty subsets of X, and all $x \in S$, we define
$U(x, S, q) = \{a \in S : aqx\}$, and
$L(x, S, q) = \{a \in S : xqa\}$.

Then, for all $S \in \chi$, define $G(S, q)$ as

$$(**) \begin{cases} \{x \in S : |U(x, S, q)| = |L(x, S, q)|\}, \text{ if } |S| \text{ is odd,} \\ \{x \in S : |U(x, S, q)| - 1 = |L(x, S, q)|\}, \text{ if } |S| \text{ is even.} \end{cases}$$

$|S|$ stands for the cardinality of set S. Note that in the case of an even number of objects, there are two median elements "theoretically". In the original Gaertner–Xu paper we defined both these elements as median elements. In the present version, a decision in the sense of uniqueness is made, defining the right of the two objects as the median element. We now wish to say that a choice function is median–type rationalizable iff there exists a linear ordering q on X such that $C(S) = G(S, q)$, for all $S \in \chi$.

Again we do not present the original set of axioms put forward by Gaertner and Xu (1999), but use the recent set of axioms from Xu (2007).

Definition 13 – Non–Emptiness of Singleton Choice Situations (NESCS). For all $x \in X, C(\{x\}) = \{x\}$.

Definition 14 – Independence of Rejected Alternatives (IRA). For all $S \in \chi$ with $\#S \geq 3$, there exist distinct $x, y \in S$ such that $C(S) \cap \{x, y\} = \emptyset$ and $C(S') = C(S' - \{x, y\})$ for all $S' \subseteq S$.

Definition 15 – Minimal Consistency of Rejection (MCR). For all distinct $x, y, z \in X$, if $\{x\} \neq C(\{x, y, z\})$ and $\{x\} \neq C(\{x, y\})$, then $\{x\} \neq C(\{x, z\})$.

These three axioms together with two earlier ones from section 2.2 will give us our result. But before that, let us briefly explain the new axioms.

Axiom NESCS says that the choice from a singleton set is always the single element. Axiom IRA states that in the case of at least three alternatives, there is always a pair of alternatives that is choice–irrelevant (i.e. each of them is never chosen) and this also holds for all subsets of the considered set. Axiom MCR specifies that if x is rejected from a triple of alternatives, it is also rejected in pairwise contests.

Here is the characterization result given by Xu (2007).

Result 6. *A choice function C that reflects the norm of picking the median element is in this sense rationalizable iff it satisfies axioms NESCS, NENCS, AE, IRA and MCR.*

We see that the characterization results 5 and 6 have two axioms in common, and it is the anti–expansion condition in particular to which we want to draw the reader's attention. We mentioned earlier that the maxim of picking the median element neither satisfies the latter condition nor Manzini and Mariotti's Weak WARP axiom. The issue of pathology comes up again. By no means do we want to claim that the two rules from the present and preceding section are THE rules to follow. But making a balanced choice is a deeply rooted principle that looks for the center of gravity, as advocated by the Confucian school. So many people seem to have internalized it in order to apply it under various circumstances. And, fortunately, many people follow certain rules of modesty and politeness.

3 Some Other Non–Standard Choice Functions

Does non–standard choice "automatically" mean that one of the standard rationality or consistency conditions is violated? Not necessarily. Consider, for example, the so–called "Mother Teresa" choice function.

If q again stands for the relevant "quality"–ordering over a given set of objects, let's say that q orders the elements according to their size or weight or value, a person who always picks the lowest or smallest object according to q perfectly satisfies the standard consistency condition. WARP and Arrow's consistency requirement are "easily" fulfilled.

Consider a choice procedure where at stage 1, the largest and the smallest elements according to ordering q are eliminated, and where at stage 2 the then largest element is picked. This two–stage choice procedure does not satisfy Arrow–consistency as can be seen from the following example. Let $S = \{a, b, c, d, e, f, g\}$ with a being largest and g being smallest according to q. Then elements a and g are deleted at stage 1 and b is picked at stage 2. Next, consider $S' = \{b, c, d, e, f\}$. Elements b and f are now deleted at stage 1 and c is chosen at stage 2. Clearly, b which was chosen from S is still available under S' but not picked.

Recently, Salant and Rubinstein (2008) considered various choice functions "with frames" which they call extended choice functions. A frame represents "observable information that is irrelevant in the rational assessment of the alternatives, but nonetheless affects choice" (2008, p. 1287). A frame can be a status–quo bias, an aspiration level or a deadline for making a choice. In the latter case, the decision maker needs time to process the information relevant to each alternative available for choice. However, a deadline has been set before which the choice has to be performed. In the sequel, we consider a choice procedure which is based on two orderings, viz. an attention ordering and a preference ordering.

The attention ordering O with respect to X, the set of alternatives, determines the objects the decision maker focuses on. The preference ordering P represents the decision maker's preferences. Given (A, n) where n is the number of objects the decision maker can actually consider, the decision maker chooses the best element according to P among the first min $\{n, |A|\}$ elements in A based on ordering O.

Salant and Rubinstein formulate three properties that characterize the class of all extended choice functions $C_{o,p}(A, n)$.

Definition 16 – Attention 1. If $C(\{a, b\}, 1) = a$, then for every set A that contains a and b, $C(A, 1) \neq b$.

This property says that if alternative a is more accessible to the decision maker than object b, then for every set A that contains a and b, the decision maker will not pick b if the amount of attention devoted to the choice problem is small.

Definition 17 – Attention 2. If $C(\{a, b\}, 2) = a$, then for every set A, $C(A, |A|) \neq b$.

This property states that if object a is picked when the decision maker considers both a and b, then for every set A that contains a and b, the decision maker will not choose b when he considers all the elements of set A.

Definition 18 – Attention 3. If $C(A, k) = a$ and $C(\{x, y\}, 1) = y$ for every $y \in A$, then $C(A \cup \{x\}, k) = a$.

This property says that adding an element x to a set A, where x is less accessible than all the elements of A, does not alter the choice as long as the amount of attention k, devoted to the deliberation process remains constant.

Note that for this class of extended choice functions, the two relations P and O are treated completely symmetrically though P expresses the preferences of the decision maker and O merely reflects the degree of attention devoted to the choice procedure. Note also that this class of choice functions violates Arrow's consistency condition. If for three elements x, y and z, we have $xOyOz$ but $zPyPx$, then $C(\{x, y, z\}, 2) = y$, while $C(\{y, z\}, 2) = z$.

4 Concluding Remarks

In the foregoing sections, we discussed a larger number of – what we have called – non–standard choice functions which except for the Mother Teresa choice function have the property that standard consistency conditions are violated. Are simple mistakes or a high degree of complexity of the underlying options the reason for this? Not at all. We argued that all these choice functions appear reasonable under specified circumstances.

We would like to make it very clear that standard utility maximization plays a very productive role in certain environments, for example in consumer theory when commodities are narrowly defined. If what counts are the characteristics of certain consumer goods, characteristics which are clearly identifiable and measurable and not properties such as the last apple in a fruit basket or the largest or the most gorgeous piece of cake or the shiniest chair at a garden–party, then ordinary utility maximization can be the appropriate tool. This is so because the external reference is confined to the derivation of a maximum amount of "utils" generated by different combinations of commodity characteristics. We would never claim, for example, that choosing the median element in such situations makes a lot of sense. However, when individual–based internalized norms or societal norms play an important role, the picture can change drastically as we have seen. Unfortunately, since there exists a multitude of norms of different kinds, it appears highly unlikely that a larger set of properties will be satisfied by *all* choice functions based on norms (see Baigent (2007)).

At the end of this paper, we would like to add one more thought. We have described the individual alternatives for choice in a narrow sense. So if one set of objects contains two identically looking apples and the second set just contains one of these apples and not the other, we have "tacitly" assumed that the latter is a strict subset of the former. Consider the following situation where the host of the evening presents a basket S containing two identical pears and two identical apples to one of his guests, i.e., $S = \{p, p; a, a\}$. Assume that the guest picks one of the apples so that $\{a\} = C(S)$. Now reduce the fruit basket from S to S' such that $S' = \{p, p; a\}$. If the guest now picks a pear, i.e., $\{p\} = C(S')$, Arrow's contraction consistency condition is violated since $a \in S' \subset S, a \in C(S)$, but $\{p\} = C(S')$. If, however, the last or only apple in set S' is considered to be essentially different from one of the two identically looking apples in S, we do not have $S' = \{p, p; a\}$ but $S'' = \{p, p; \hat{a}\}$ so that it is no longer true that $S'' \subset S$. Object $\hat{a}$ is different from $a \in S$ since $\hat{a}$ is the only apple left in the fruit basket. If alternatives are defined in such a comprehensive way, one can argue that for a given set S, there are no proper subsets at all. Each set S' which no longer contains one or several elements from S is a set of its own, with new characteristics so to speak. Apple a as the last remaining apple in the fruit basket and apple a among several identical apples are no longer the same. Clearly, in such a world, contraction and expansion consistency conditions are trivially satisfied. Comprehensively describing alternatives or states of the world can be done, if at all, without any clearly defined limits but to say the least, this is not the path that general economic analysis pursued in the past (see also Baigent and Gaertner (1996), p. 241).

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